Lesson 17: Combined Mode Control

ET 438a Automatic Control Systems Technology

Learning Objectives

After this presentation you will be able to:

- Describe the common control mode combinations used in analog control systems.
- List the characteristics of combined control modes.
- Write the time, Laplace and transfer functions of combined control modes.
- Identify the Bode plots of combined control modes.
- Design OP AMP circuits that realize theoretical combined control mode performance.

Proportional-Integral Control

Control Mode Characteristics:

1. Proportional action produces fast response to large load changes.
2. Integral action drives output to zero steady-state error.
3. Adds one pole and one zero to system transfer function.
4. Used on systems that have large load changes and where proportional only action fails to reduce steady-state error to acceptable limits.

Proportional-Integral Control

Mathematical Representations

Time Function: $v(t) = K_p \cdot e(t) + K_p \cdot K_I \cdot \int_0^{\tau} e(t)\,dt + v_0$

Laplace Function: $V(s) = K_p \cdot E(s) + \left[\frac{K_p \cdot K_I}{s}\right] \cdot E(s)$

Transfer Function: $\frac{V(s)}{E(s)} = K_p \cdot \left[\frac{K_I + s}{s}\right]$

All initial conditions set to zero in transfer function

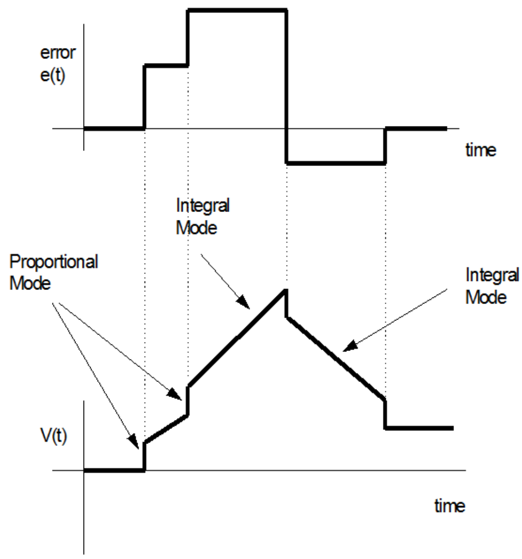
zero

pole

Proportional-Integral (PI) Controllers add a pole at s=0 and zero at s=0 to system transfer function

Proportional-Integral Control Response

Time plots of PI controller output for piece-wise linear error input

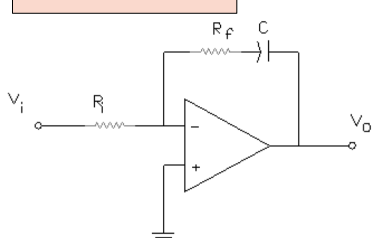


Proportional mode gives instantaneous response to error while integral mode decrease error over time.

Constant error produces linearly increasing controller output

OP AMP Realization of PI Controller

PI Controller Circuit



$$K_p = \frac{R_f}{R_i} \qquad K_I = \frac{1}{R_i \cdot C}$$

Derive the transfer function from the generalized gain formula for inverting OP AMP circuits

$$A_V(s) = \frac{-Z_f(s)}{Z_i(s)}$$

$$Z_i(s) = R_i \qquad Z_f(s) = R_f + \frac{1}{C \cdot s}$$

$$A_V(s) = \frac{-\left[R_f + \frac{1}{C \cdot s}\right]}{R_i} = \frac{-(C \cdot s)\left[R_f + \frac{1}{C \cdot s}\right]}{(C \cdot s)R_i}$$

$$A_V(s) = \frac{-\left[R_f \cdot C \cdot s + \frac{C \cdot s}{C \cdot s}\right]}{R_i \cdot C \cdot s} = \frac{-[R_f \cdot C \cdot s + 1]}{R_i \cdot C \cdot s}$$

Controller introduces a pole at s=0 and zero at s=-1/R_fC

Simplify $$A_V(s) = -\left[\frac{R_f \cdot C \cdot s}{R_i \cdot C \cdot s} + \frac{1}{R_i \cdot C \cdot s}\right] = \frac{-R_f}{R_i} + \frac{-1}{R_i \cdot C \cdot s}$$

Bode Plot of PI Controller

Let R_i = 10 kΩ, R_f = 100 kΩ and C=0.01 μF then produce Bode plot

MatLAB Code

```
ri=input('Enter value of input resistance: ');
c=input('Enter value of capacitance: ');
rf=input('Enter value of feedback resistance: ');
% compute transfer function model parameters for
% PI controller
% compute numerator parameter
tau=rf*c;
% compute parameter for denominator
tau1 = ri*c;
% build transfer function
% denominator form a1*s^2+a2s+a3
Av=tf([tau 1],[tau1 0])

%plot graph
bode(Av);
```

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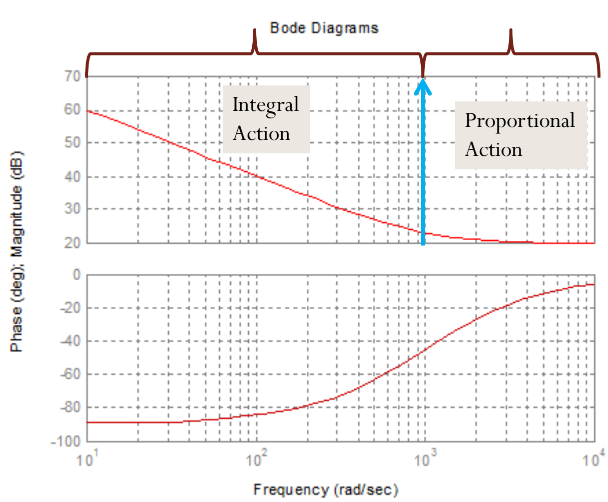
Bode Plot of PI Controller



Integral action is below 1000 rad/sec.

$1/R_fC$ set break point

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PI Controller Design

Example 17-1: Design a PI OP AMP controller with $K_p = 100$ and an integral break frequency of 100 rad/sec. $R_i = 10k\Omega$

$$K_p = \frac{R_f}{R_i} \quad \omega = \frac{1}{R_f \cdot C} \quad K_p = 100 \quad R_i = 10{,}000\ \Omega$$

$$K_p = \frac{R_f}{R_i} \quad \Rightarrow R_f = R_i \cdot K_p = (10{,}000\ \Omega)(100) = 1{,}000{,}000\ \Omega$$

$$R_f = 1.0\ M\Omega$$ Answer

$$\omega = 100 \text{ rad/sec}$$

$$\omega = \frac{1}{R_f \cdot C} \quad \Rightarrow C = \frac{1}{R_f \cdot \omega}$$

$$C = \frac{1}{(1.0\ M\Omega)(100 \text{ rad/sec})} = 1\times10^{-8}\ F$$

$$C = 0.01\ \mu F$$ Answer

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Proportional-Derivative Control Mode

Proportional-Derivative (PD) Control combines proportional and derivative actions. Used in processes that have sudden load changes that proportional only cannot handle.

Mathematical Representations

Time Function:

$$v(t) = K_p \cdot e(t) + K_p \cdot K_d \cdot \frac{d\,e(t)}{dt} - \alpha \cdot K_d \cdot \frac{d\,v(t)}{dt} + v_0$$

$$\alpha \cdot \frac{d\,v(t)}{dt} = \text{Rate limit to high frequencies}$$

Laplace Function:

$$V(s) = K_p \cdot E(s) + K_p \cdot K_d \cdot s \cdot E(s) - \alpha \cdot K_d \cdot s \cdot E(s)$$

Transfer Function:

$$\frac{V(s)}{E(s)} = K_p \cdot \left[\frac{1 + K_d \cdot s}{1 + \alpha \cdot K_d \cdot s}\right]$$

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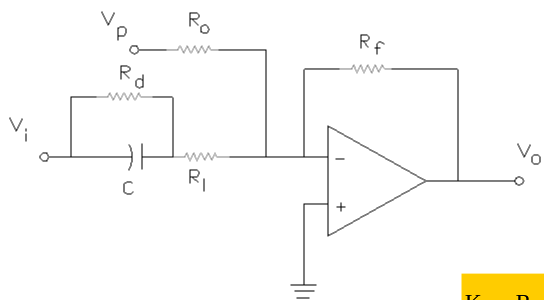
OP AMP Realization of PD Controller

OP AMP PD circuit



$$\frac{V(s)}{E(s)} = K_p \cdot \left[\frac{1 + K_d \cdot s}{1 + \alpha \cdot K_d \cdot s}\right]$$

$K_d = R_d \cdot C$ $\alpha = \frac{R_I}{R_I + R_d}$ $K_p = \frac{R_f}{R_I + R_d}$

$R_0 = R_f$

Note: $1/K_d$ = the derivative action break point frequency
$1/\alpha K_d$ = limiter action break point frequency
$0 \le \alpha \le 1$

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PD Controller Design

<u>Example 17-2:</u> Design a PD control that has a proportional gain of 10 a derivative action break point of 100 rad/sec and a limiter frequency break point of 1000 rad/sec. C = 0.1 μF. Draw the OP AMP circuit and place all values on the schematic.

Example 17-2 Solution

$K_p = 10$ $\frac{1}{K_d} = 100 \text{ rad/sec}$ $\frac{1}{\alpha K_d} = 1000 \text{ rad/sec}$

$R_d C = K_d$ so $\frac{1}{R_d C} = \frac{1}{K_d}$ $C = 0.1 \times 10^{-6}$

$$\frac{1}{R_d(0.1 \times 10^{-6})} = \frac{1}{100}$$

$$R_d = \frac{1}{(0.1 \times 10^{-6})(100)} = \boxed{100\,k\Omega}$$

Answer

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Example 17-2 Solution (2)

Now compute the alpha value from the rate limit break frequency

$$\frac{1}{\alpha K_d} = 1000 \text{ rad/sec} \qquad \frac{1}{K_d} = 100 \text{ rad/sec} \rightarrow K_d = 0.01$$

$$1 = (1000 \text{ rad/sec})\alpha K_d$$

$$\frac{1}{1000 \text{ rad/sec } K_d} = \alpha$$

$$\frac{1}{(1000 \text{ rad/sec})(0.01)} = \alpha$$

$$\boxed{\alpha = 0.1}$$ ← Answer

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Example 17-2 Solution (3)

Now find the value of R_I

$$\alpha = \frac{R_I}{R_I + R_d}$$

From previous calculations $R_d = 100\ \text{k}\Omega$ and $\alpha = 0.1$

$$0.1 = \frac{R_I}{R_I + 100\ \text{k}\Omega}$$

$$0.1(R_I + 100{,}000) = R_I$$

$$0.1 R_I + 10{,}000 = R_I$$

$$10{,}000 = 0.9 R_I$$

$$\frac{10{,}000}{0.9} = R_I$$

$$\boxed{11{,}111\ \Omega = R_I}$$ ← Answer

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Example 17-2 Solution (4)

Find R_f and R_o use proportional gain and previously computed values

$$\frac{R_f}{R_I + R_d} = K_p$$

Substitute values

$$\frac{R_f}{11{,}111 + 100{,}000} = 10$$

$$R_f = 10(11{,}111 + 100{,}000)$$

$$\boxed{R_f = 1.11\ M\Omega}$$

Answer

$$R_f = R_o = 1.11\ M\Omega$$

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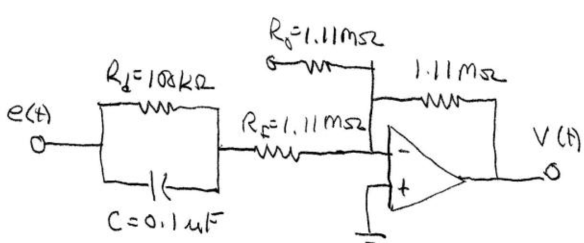
Example 17-2 Solution (5)

Draw OP AMP Schematic and label components



Transfer function

$$\frac{V(s)}{E(s)} = -K_p\left[\frac{1+K_d s}{1+\alpha K_d s}\right] = -10\left[\frac{1+0.01s}{1+0.001s}\right]$$

Negative sign causes 180 degree phase shift due to inverting configuration. Add Inverting OP AMP circuit with gain of -1 to remove this

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PD Bode Plot

Use MatLAB to generate the Bode plot of the PD controller designed in Example 17-2

MatLAB Code

Define variables

```
ri=11.11e3;
c=0.1e-6;
rf=1.111e6;
rd=1e5;
```

Compute parameters

```
% compute transfer function model parameters for
% PD controller
% compute parameters
kd=rd*c;
alpha=ri./(ri+rd);
kp=-rf./(ri+rd);
```

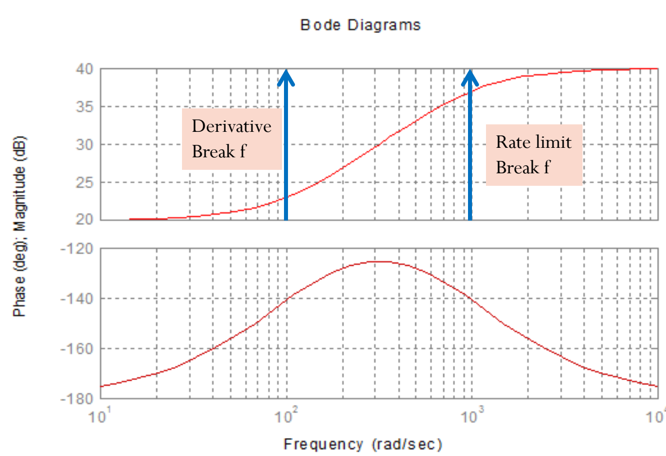
Generate transfer function and plot response

```
% build transfer function
% denominator form a1*s^2+a2s+a3
% numerator for b1*s^2+b2s+b3
Av1=kp*tf([kd 1],[alpha*kd 1])
%plot graph
bode(Av1);
```


PD Bode Plot

Computed transfer function

```
-0.1 s - 10
---------------
 0.001 s + 1
```



Proportional + Integral + Derivative Controllers

Proportional-Integral-Derivative (PID) Controller Characteristics:

1. Proportional mode provides fast response to large process load changes
2. Integral mode removes the steady-state (offset) error from the output
3. Derivative mode improves system stability and improves response to rapid load changes.
4. Widely used in industry

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Mathematical Relationships for PID Controllers

Time Function:

$$v(t) = K_p \cdot e(t) + K_p \cdot K_I \cdot \int_0^{\tau} e(t)\,dt + K_p \cdot K_d \cdot \frac{d\,e(t)}{dt} - \alpha \cdot K_d \cdot \frac{d\,v(t)}{dt} + v_0$$

Laplace Function:

$$V(s) = K_p \cdot E(s) + \left[\frac{K_p \cdot K_I}{s}\right] \cdot E(s) + K_p \cdot K_d \cdot s \cdot E(s) - \alpha \cdot K_d \cdot s \cdot E(s)$$

$$V(s) = K_p \cdot \left[E(s) + \left[\frac{K_I}{s}\right] \cdot E(s) + K_d \cdot s \cdot E(s)\right] - \alpha \cdot K_d \cdot s \cdot E(s)$$

Transfer Function:

$$\frac{V(s)}{E(s)} = K_p \cdot \left[\frac{K_I + s + K_d \cdot s^2}{s + \alpha \cdot K_d \cdot s^2}\right]$$

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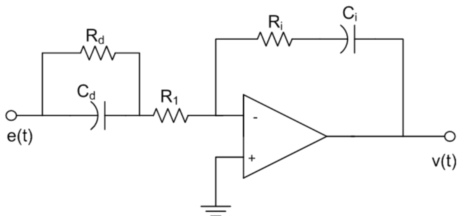
OP AMP Realization of PID Controller



$$\frac{V(s)}{E(s)} = -K_p \cdot \left[\frac{K_I + s}{s}\right] \cdot \left[\frac{1 + K_d \cdot s}{1 + \alpha \cdot K_d \cdot s}\right]$$

$$\frac{V(s)}{E(s)} = -K_p \cdot \left[\frac{K_I + (K_I \cdot K_d + 1) \cdot s + K_d \cdot s^2}{s + \alpha \cdot K_d \cdot s^2}\right]$$

$$K_p = \frac{R_i}{R_i + R_1}$$

$$\alpha = \frac{R_1}{R_1 + R_d}$$

$$K_d = R_d \cdot C_d$$

$$K_I = \frac{1}{R_i \cdot C_i}$$

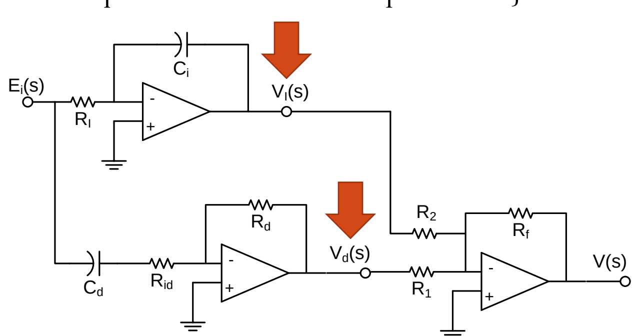
Single OP AMP realization of PID control

Modify system response by changing the values of K_p, K_d, K_I and α.

Notice that the parameters are dependent on each other. Changing alpha will also change K_p.

OP AMP Realization of PID Controller

Multiple OP AMPs allow independent adjustment of PID parameters.



$$\frac{V_I(s)}{E(s)} = \frac{-1}{R_I \cdot C_I \cdot s}$$

$$\frac{V_d(s)}{E(s)} = \frac{-R_d \cdot C_d \cdot s}{1 + R_{id} \cdot C_d \cdot s}$$

$$\frac{V(s)}{E(s)} = \frac{R_f}{R_1}\left[\frac{1}{R_I \cdot C_I} + \frac{R_d \cdot C_d \cdot s}{1 + R_{id} \cdot C_d \cdot s}\right]$$

Where $R_1 = R_2$

OP AMP Realization of PID Controller

Multiple OP AMP Realization

Compute common denominator

$$\frac{V(s)}{E(s)} = \frac{R_f}{R_1}\left[\frac{1}{R_I C_I s} + \frac{R_d C_d s}{1 + R_{id} C_d s}\right]$$

$$\frac{V(s)}{E(s)} = \frac{R_f}{R_1}\left[\frac{1 + R_{id} C_d s}{R_I C_I s(1 + R_{id} C_d s)} + \frac{R_d C_d s(R_I C_I s)}{1 + R_{id} C_d s(R_I C_I s)}\right]$$

$$\frac{V(s)}{E(s)} = \frac{R_f}{R_1}\left[\frac{1 + R_{id} C_d s}{R_I C_I s + +R_{id} R_I C_d C_I s^2} + \frac{R_d C_d R_I C_I s^2}{R_I C_I s + +R_{id} R_I C_d C_I s^2}\right]$$

Define the following relationships between components and parameters

$$\frac{1}{K_I} = R_I C_I \qquad K_d = R_d C_d \qquad K_p = \frac{R_f}{R_1} \qquad \alpha K_d = R_{id} C_d$$

Multiple OP AMP PID Realization

Divide numerator and denominator by $R_I C_I$ and make defined substitutions

$$\frac{V(s)}{E(s)} = \frac{R_f}{R_1}\left[\frac{\frac{1}{R_I C_I} + \left(\frac{1}{R_I C_I}\right) R_{id} C_d s + \left(\frac{1}{\cancel{R_I C_I}}\right) R_d C_d \cancel{R_I C_I} s^2}{\left(\frac{1}{\cancel{R_I C_I}}\right)\cancel{R_I C_I} s + \left(\frac{1}{\cancel{R_I C_I}}\right) R_{id} \cancel{R_I} C_d \cancel{C_I} s^2}\right]$$

$$\frac{V(s)}{E(s)} = K_p\left[\frac{K_I + K_I \alpha K_d s + K_d s^2}{s + \alpha K_d s^2}\right]$$

Controller adds two zeros and two poles to System. Change system response by changing Parameters K_p K_I, K_d α.

Where

$$\frac{1}{K_I} = R_I C_I \qquad K_d = R_d C_d \qquad K_p = \frac{R_f}{R_1} \qquad \alpha K_d = R_{id} C_d$$

PID Controller Design

Example 17-3: Design a PID controller using the circuit above. Proportional gain is 5. The derivative time constant is 0.5 seconds. The integral gain is 0.143 and α=0.1. The capacitor for the integrator C_i=10 μF and the differentiator, C_d= 1 μF R_f = 1 MΩ. Find values for all other components.

Example 17-3 Solution

Define parameters

$K_p = 5$ $\quad C_d = 1\ \mu F$ $\quad K_I = \frac{1}{R_i C_i}$

$K_I = 0.143$ $\quad R_{fd} = 1\ M\Omega$

$\alpha = 0.1$ $\quad C_i = 10\ \mu F$

$K_d = 0.5$

$$R_i = \frac{1}{K_I C_I} = \frac{1}{(0.143)(10\ \mu F)} = 700,000$$

$R_i = 700,000$ Answer

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Example 17-3 Solution (2)

Find the feedback resistor value for the differentiator

$K_d = 0.5$ $\quad K_d = R_d C_d$

$C_d = 1\ \mu F$

$$\frac{K_d}{C_d} = R_d \quad \frac{0.5}{1 \times 10^{-6} F} = 500,000\ \Omega = R_d$$

Answer

Now find the input resistor values for the summing amplifier

$K_p = 5$ $\quad \frac{R_f}{R_1} = K_p$ $\quad R_1 = \frac{1 \times 10^6}{5} = 200,000$

$R_f = 1\ M\Omega$ $\quad R_1 = \frac{R_f}{K_p}$ $\quad R_1 = R_2 = 200,000\ \Omega$ Answer

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Example 17-3 Solution (3)

Find the input resistor value for the differentiator

$\alpha = 0.1$

$\alpha K_d K_I = R_{id} C_d \qquad C_d = 1\,\mu F$

Solve for R_{id} and substitute in all known values

$$\frac{\alpha K_d K_I}{C_d} = R_{id}$$

$$\frac{0.1(0.5)(0.143)}{1\times10^{-6}} = R_{id}$$

$$7150\,\Omega = R_{id}$$

Answer

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End Lesson 17: Combined Mode Control

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